Multi-brid inflation and non-Gaussianity

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We consider a class of multi-component hybrid inflation models whose evolution may be analytically solved under the slow-roll approximation. We call it multi-brid inflation (or n-brid inflation where n stands for the number of inflaton fields). As an explicit example, we consider a two-brid inflation model, in which the inflaton potentials are of exponential type and a waterfall field that terminates inflation has the standard quartic potential with two minima. Using the δN formalism, we derive an expression for the curvature perturbation valid to full nonlinear order. Then we give an explicit expression for the curvature perturbation to second order in the inflaton perturbation. We find that the final form of the curvature perturbation depends crucially on how the inflation ends. Using this expression, we present closed analytical expressions for the spectrum of the curvature perturbation $\mathcal{P}_S(k)$, the spectral index n_S, the tensor to scalar ratio r, and the non-Gaussian parameter $f_{NL}^{\rm local}$, in terms of the model parameters. We find that a wide range of the parameter space $(n_S,\ r,\ f_{NL}^{\rm local})$ can be covered by varying the model parameters within a physically reasonable range. In particular, for plausible values of the model parameters, we may have a large non-Gaussianity $f_{NL}^{\rm local} \sim 10$–$100$. This is in sharp contrast to the case of single-field hybrid inflation in which these parameters are tightly constrained.

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I. INTRODUCTION

The standard, single-field slow-roll inflation predicts the curvature perturbations which are almost Gaussian to high accuracy [1, 2]. Thus the detection or non-detection of non-Gaussianity will have extremely important implications to theories of the early universe, and a variety of multi-component models that give detectable non-Gaussianities have been proposed [3–10]. In many of these cases studied so far, however, it is not quantitatively clear how the non-Gaussianity arises and what determines its level.

In this paper, we study a class of multi-component hybrid-type inflation models whose dynamics can be analytically solved. Similar exactly soluble slow-roll models were investigated in Ref. [11]. Here we extend the analysis given in Ref. [11] by including a coupling to a water-fall field which terminates the inflationary stage. Then applying the δN formalism [12–15], we compute the curvature perturbation analytically. Models of hybrid inflation similar to the one presented in this paper were proposed and investigated by Bernardeau and Uzan [5] and by Alabidi and Lyth [9].

In the δN-formalism, the final amplitude of the curvature perturbation on comoving hypersurfaces $\mathcal{R}_c$ (or equivalently on uniform total density hypersurfaces ζ) is given by δN, where δN is the perturbation of the number of e-folds between the initial flat time-slice at horizon crossing during inflation and a comoving time-slice during the final radiation-dominated stage by which time all the inflationary trajectories are assumed to have converged to a unique one [13, 15].

For the models we consider in this paper, assuming the scalar field perturbations are Gaussian up to the time of horizon crossing, the non-Gaussianity is local in the sense that the curvature perturbation at each spatial point is expressed in terms of a nonlinear function of Gaussian fields at the same spatial point. In this case, the level of non-Gaussianity is conveniently characterized by the quantity $f_{NL}^{\rm local}$ [16].

This paper is organized as follows. In Section II, we first consider conditions for a slow-roll model to be exactly soluble and derive a general formula for the curvature perturbation. In Section III, as a specific example, we present a model of multi-field hybrid inflation, which we call "*multi-brid inflation*". Then focusing on the case of two-brid inflation, we give various formulas explicitly. In Section IV, we compute analytically the spectrum of the curvature perturbation $\mathcal{P}_S(k)$, its spectral index n_S, the tensor-to-scalar ratio r, and the non-Gaussian parameter $f_{NL}^{\rm local}$. They are all expressed explicitly in terms of the model parameters. We conclude the paper in Section V. Some computational details are described in Appendix A. We use the Planck units where $M_{pl}^{-2} = 8\pi G = 1$.

II.   EXACT SOLUBLE CLASS

We consider an Einstein-scalar Lagrangian,

$$L = \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}h_{ab}(\phi)\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\,, \tag{2.1}$$

where R is the Ricci scalar, the Latin indices a, b run from 1 to M, h_{ab} is the field space metric, and V is the potential. We assume the dynamics of inflation is described by this Lagrangian. Later we introduce a water-fall field χ that terminates inflation. For the moment, however, we concentrate our discussion on the inflationary stage.

The Friedmann equation and the field equations are, respectively,

$$3H^2 = \frac{1}{2}h_{ab}\dot\phi^a\dot\phi^b + V(\phi)\,,$$
$$\ddot\phi^a + 3H\dot\phi^a + h^{ab}\partial_b V = 0\,, \tag{2.2}$$

where $H = \dot a/a$ with a being the cosmic scale factor, and a dot denotes a derivative with respect to the cosmic proper time t, $\dot{} = d/dt$. The slow-roll equations of motion are obtained by neglecting the kinetic term in the Friedmann equation and the second time derivative in the field equation,

$$3H^2 = V(\phi)\,,$$
$$3H\dot\phi^a + h^{ab}\partial_b V = 0\,. \tag{2.3}$$

For later convenience, we change the time variable from t to the number of e-folds from the end of inflation *backward* in time,

$$dN = -H\,dt\,. \tag{2.4}$$

Then the slow-roll equations of motion give

$$\frac{d\phi^a}{dN} = \frac{h^{ab}\partial_b V}{3H^2} = \frac{h^{ab}\partial_b V}{V}\,. \tag{2.5}$$

Now we consider the case when the slow-roll equations of motion (2.5) can be exactly soluble. They can be exactly solved if the right-hand side of Eq. (2.5) for each index a takes the form,

$$\frac{h^{ab}\partial_b V}{V} = \frac{f^a(\phi^a)}{F(\phi)}\,, \tag{2.6}$$

where F represents an arbitrary function of $(\phi^1, \phi^2, \cdots, \phi^M)$, and f^a is a function of only ϕ^a for each a ($a = 1, 2, \cdots, M$). In this case, we have

$$\frac{1}{f^a(\phi^a)}\frac{d\phi^a}{dN} = \frac{1}{F(\phi)}\,. \tag{2.7}$$

Introducing a new set of field coordinates q^a by

$$\ln q^a = \int \frac{d\phi^a}{f^a}\,, \tag{2.8}$$

the equations of motion become

$$\frac{d\ln q^a}{dN} = \frac{1}{F}\,. \tag{2.9}$$

Here we introduce the radial and angular coordinates of the field space,

$$q^a = q\,n^a\,; \quad \sum_a (n^a)^2 = 1\,. \tag{2.10}$$

Then it is straightforward to see that n^a is conserved,
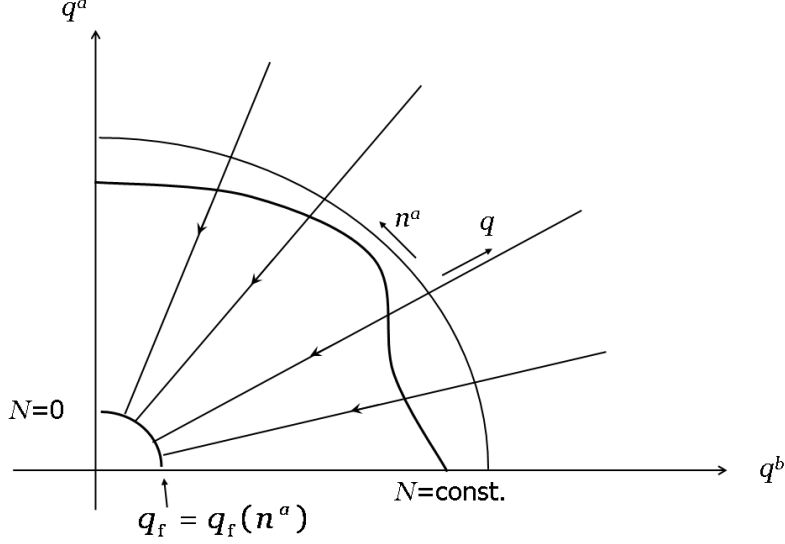
$$\frac{dn^a}{dN} = 0\,, \tag{2.11}$$

FIG. 1: A schematic diagram of classical trajectories in the field space with the coordinates q^a. The angular coordinates $n^a = q^a/q$ are conserved, and hence all the trajectories are radial in these coordinates. The curve indicated by $q = q_f$ denotes the surface at which the inflation ends, which may depends on n^a. The e-folding number N is counted backward in time from $q = q_f$.

and Eq. (2.9) reduces to a single equation for q,

$$\frac{d \ln q}{dN} = \frac{1}{F(q, n^a)} \,, \tag{2.12}$$

where F is now regarded as a function of q and $(n^1, n^2, \cdots, n^M)$.

In fact, what we have shown is not restricted to slow-roll inflation. Whenever a system approaches an attractor-like asymptotic stage, the value of the field at one instant of time completely determines the motion of the field at its subsequent time. In such a case, we have M first-order differential equations instead of M second-order differential equations. Then the exact solubility means there are $M-1$ constants of integration. This is indeed the case we have considered in the above (there are $M-1$ degrees of freedom in n^a). A visualization of this situation is depicted in Fig. 1.

At this point, let us consider a couple of somewhat more specific examples. The condition for exact solvability (2.6) is satisfied if one can choose a set of field space coordinates such that the field space metric takes the form,

$$h^{ab} = \frac{\text{diag.} \left(h^1(\phi^1), h^2(\phi^2), \cdots, h^n(\phi^n) \right)}{H(\phi)} \,, \tag{2.13}$$

and the potential is ether of product type,

$$V = \prod_a V^a(\phi^a) \,, \tag{2.14}$$

in which case we have

$$f^a = h^a \frac{d \ln V^a}{d\phi^a} \,, \quad F = H \,. \tag{2.15}$$

or of separable type,

$$V = \sum_a V^a(\phi^a) \,, \tag{2.16}$$

in which case we have

$$f^a = h^a \frac{dV^a}{d\phi^a}, \quad F = HV. \tag{2.17}$$

The separable case was first investigated in detail by Starobinsky [17].

Now, going back to the general case, Eq. (2.11) implies the trajectories are always radial in the field space spanned by q^a, and Eq. (2.12) can be easily solved for N to give

$$N(q, n^a) = \int_{q_f}^q F(q', n^a) \, d\ln q', \tag{2.18}$$

where q_f is the value of q at the end of inflation. Note that q_f generally depends on n^a. Thus, applying the nonlinear δN formula, we immediately obtain a fully nonlinear expression for the conserved comoving curvature perturbation,

$$\delta N = N(q + \delta q, n^a + \delta n^a) - N(q, n^a). \tag{2.19}$$

It is instructive to consider the linear limit of the above δN. Denoting it by $\delta_L N$, we find

$$\delta_L N = F(q, n^a) \frac{\delta q}{q} + \int_{q_f}^q \frac{\partial F}{\partial n^a} \frac{dq'}{q'} \delta n^a - \frac{F}{q_f} \frac{\partial q_f}{\partial n^a} \delta n^a, \tag{2.20}$$

where the values of (q, n^a) are those at horizon crossing during inflation and $(\delta q, \delta n^a)$ are the fluctuations evaluated on the flat hypersurface at that epoch. It may be noted that δq^a are given by the fluctuations of the original field variables $\delta\phi^a$ as

$$\delta \ln q^a = \frac{\delta\phi^a}{f^a(\phi^a)}. \tag{2.21}$$

The formula (2.20) consists of three terms. The first term may be interpreted as the one due to adiabatic perturbations, the second to entropy perturbations during inflation [13, 18, 19], and the third to entropy perturbations at the end of inflation [6–8]. It may be worthwhile to mention that although these distinctions are meaningful and useful at linear order (or perhaps perturbatively), they are not so at full nonlinear order.

Figure 2 is a schematic graph to explain the meaning of these three terms. The thick lines represent three different kinds of trajectories in the field space, and the thin lines with arrows on both ends represent the field fluctuations. The wavy dashed line represents the surface at which inflation ends. After inflation, each kind of orbit converges to a unique trajectory.

The one on the left, where the fluctuations are parallel to the orbits, corresponds to the first term in Eq. (2.20). In this case, the fluctuations in the initial condition directly gives δN. This is identical to the conventional adiabatic curvature perturbation in the single field case. The one in the center corresponds to the second term. The fluctuations are orthogonal to the orbits, so they are entropy (isocurvature) perturbations during inflation. However, by the time the two adjacent orbits converge to a unique orbit, one finds the number of e-folds depends on which route the universe has taken. This gives rise to δN before the end of inflation. The one on the right corresponds to the third term. The entropy perturbations are not converted to δN until the end of inflation. However, the surface that determines the end of inflation may not be orthogonal to the orbits. This gives rise to δN in the end.

III.   MULTI-BRID INFLATION

We consider a multi-component inflaton field with the potential,

$$V(\phi) = V_0 \exp\left[\sum_{A=1}^M m_A \phi_A\right]. \tag{3.1}$$

Without loss of generality, we assume $m_A \phi_A > 0$ for all $A = 1, 2, \cdots, M$. The slow-roll equations of motion are

$$\frac{d\phi_A}{dN} = \frac{1}{V} \frac{\partial V}{\partial \phi_A} = m_A. \tag{3.2}$$
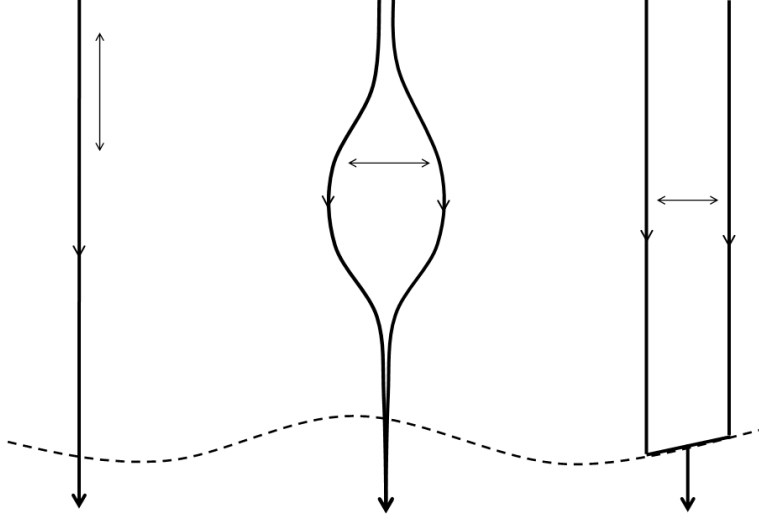
FIG. 2: Schematic graphs describing the three terms in Eq. (2.20). The thick lines represent three different kinds of orbits in the field space, and the thin lines with arrows on both ends represent the field fluctuations. The wavy dashed line represents the end of inflation. The one on the left corresponds to the first term, the one in the center to the second term, and the one on the right to the third term. See text for more details.

Note that the effective mass-squared M_A^2 for each ϕ_A is given by

$$M_A^2 = \frac{\partial^2 V}{\partial \phi_A^2} = m_A^2 V = 3\, m_A^2 H^2 \,. \tag{3.3}$$

Thus the slow-roll condition is satisfied for $m_A^2 \ll 1$. Introducing a new variable q_A by

$$q_A = \exp[\phi_A/m_A] = q\, n_A \,; \quad \sum_A (n_A)^2 = 1 \,, \tag{3.4}$$

the slow-roll equations become

$$\frac{d\ln q}{dN} = 1 \,, \quad \frac{dn_A}{dN} = 0 \,. \tag{3.5}$$

These are immediately integrated to give

$$N = \ln q - \ln q_f = \frac{1}{2} \ln \left[\sum_A e^{2\phi_A/m_A} \right] - \frac{1}{2} \ln \left[\sum_A e^{2\phi_{A,f}/m_A} \right] \,. \tag{3.6}$$

Now, similar to the conventional single-field hybrid inflation [20], we assume that inflation ends at

$$\sum_A g_A^2 \phi_A^2 = \sigma^2 \,, \tag{3.7}$$

and the universe is thermalized instantaneously. This may be realized by introducing a water-fall field χ that terminates inflation. Specifically, we promote V_0 in Eq. (3.1) to a function of χ (and of ϕ_A) as

$$V_0 = \frac{1}{2} \sum_A g_A^2 \phi_A^2 \chi^2 + \frac{\lambda}{4} \left(\chi^2 - \frac{\sigma^2}{\lambda} \right)^2 \,. \tag{3.8}$$

Then we see that the field χ is stable at the origin for $\sum_A g_A^2 \phi_A^2 > \sigma^2$, but becomes unstable for $\sum_A g_A^2 \phi_A^2 < \sigma^2$ which brings inflation to an abrupt end.

Now we restrict our discussion to the case of two inflaton fields, that is, two-brid inflation. Then it is convenient to parametrize n_A $(A = 1, 2)$ as

$$n_1 = \cos\theta, \quad n_2 = \sin\theta. \tag{3.9}$$

Similarly, we parametrize the scalar field at the end of inflation as

$$\phi_{1,f} = \frac{\sigma}{g_1}\cos\gamma, \quad \phi_{2,f} = \frac{\sigma}{g_2}\sin\gamma. \tag{3.10}$$

Note that since $m_A\phi_A > 0$ $(A = 1, 2)$, both $m_1 g_1 \cos\gamma$ and $m_2 g_2 \sin\gamma$ are positive. Then, since θ is a constant of motion, we have

$$\ln\left[\frac{q_1}{q_2}\right] = \frac{\phi_1}{m_1} - \frac{\phi_2}{m_2} = \frac{\sigma\cos\gamma}{g_1 m_1} - \frac{\sigma\sin\gamma}{g_2 m_2}. \tag{3.11}$$

This equation may be solved for γ in terms of ϕ_1 and ϕ_2. Hence we may regard γ as a function of (ϕ_1, ϕ_2), $\gamma = \gamma(\phi_1, \phi_2)$. Therefore, the number of e-folds (3.6) may now be regarded as a function of (ϕ_1, ϕ_2),

$$N = N(\phi_1, \phi_2) = \frac{1}{2}\ln\left[\frac{e^{2\phi_1/m_1} + e^{2\phi_2/m_2}}{e^{2\sigma\cos\gamma/(g_1 m_1)} + e^{2\sigma\sin\gamma/(g_2 m_2)}}\right]. \tag{3.12}$$

It is then straightforward to obtain δN to full nonlinear order. It is given simply by

$$\delta N = N(\phi_1 + \delta\phi_1, \phi_2 + \delta\phi_2) - N(\phi_1, \phi_2). \tag{3.13}$$

Here it may be useful to clarify the origin of δN in the present model. Going back to the general expression for the linear curvature perturbation (2.20), and noting that we have $F = 1$ in the equation of motion (2.12) in the present model, we see that there is no contribution from the second term which is due to entropy perturbations during inflation, Thus the linear curvature perturbation is due to the initial adiabatic perturbation at the time of horizon crossing and to the final entropy perturbation at the end of inflation. Explicit computations of these contributions to δN are given in Appendix **??**.

Before closing this section, we note that there is a correction to be added to the above formula in the rigorous sense. It comes from the fact that the surface where the inflation ends in the field space, Eq. (3.7), is not equal to the surface of constant energy density. This means that the hot Friedmann stage of the universe starts at slightly different temperatures for different values of $(\phi_{1,f}, \phi_{2,f})$ at which inflation was terminated. This can be taken care of by introducing a correction to the number of e-folds given by Eq. (3.12). Namely, denoting the potential energy at the end of inflation by V_f, we add a correction term N_c to Eq. (3.12) as

$$N = \frac{1}{2}\ln\left[\frac{e^{2\phi_1/m_1} + e^{2\phi_2/m_2}}{e^{2\sigma\cos\gamma/(g_1 m_1)} + e^{2\sigma\sin\gamma/(g_2 m_2)}}\right] + N_c(\gamma), \tag{3.14}$$

where

$$N_c = \frac{1}{4}\ln\left[\frac{V_f}{V_0}\right] = \frac{\sigma}{4}\left(\frac{m_1}{g_1}\cos\gamma + \frac{m_2}{g_2}\sin\gamma\right). \tag{3.15}$$

In the above we have assumed that the universe has become radiation-dominated right after inflation. It is then easy to see that this correction term is negligible for sufficiently small m_1 and m_2. To be more specific, if we compare the γ-dependent terms in the original N to those in N_c, the dependence is apparently strong in the former for small m_1 and m_2 irrespective of the values of σ, g_1 and g_2. Explicit computations of δN and δN_c to second order in the field fluctuations are carried out in Appendix A.

IV. CURVATURE PERTURBATION IN TWO-BRID INFLATION

Let us evaluate the curvature perturbation explicitly. The quantities of our interest are the curvature perturbation spectrum $\mathcal{P}_S$, the spectral index n_S, tensor-to-scalar ratio r, and the non-Gaussianity f_{NL}^{local}. To evaluate these

quantities, we expand the δN formula (3.13) to second order in $\delta\phi$ for $N(\phi)$ given in (3.12). Note that $\delta\gamma$ must be expressed in terms of $\delta\phi$ with second order accuracy, using Eq. (3.11). Details are given in Appendix A. The result is

$$\delta N = \frac{\delta\phi_1\, g_1 \cos\gamma + \delta\phi_2\, g_2 \sin\gamma}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma} + \frac{g_1^2 g_2^2}{2\sigma} \frac{(m_2 \delta\phi_1 - m_1 \delta\phi_2)^2}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^3} + \cdots .\tag{4.1}$$

We assume that the scalar field fluctuations $\delta\phi_1$ and $\delta\phi_2$ are Gaussian, with the dispersion,

$$\langle \delta\phi_A \delta\phi_B \rangle_k = \left.\left(\frac{H}{2\pi}\right)^2\right|_{t_k} \delta_{AB}\,,\tag{4.2}$$

where t_k is the horizon-crossing time of the comoving wavenumber k, $k = Ha$. Then the curvature perturbation spectrum is given by

$$\mathcal{P}_S(k) \equiv \frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}}(k) = \frac{g_1^2 \cos^2\gamma + g_2^2 \sin^2\gamma}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^2} \left.\left(\frac{H}{2\pi}\right)^2\right|_{t_k} .\tag{4.3}$$

Using the fact that $3H^2 = V$, the spectral index is easily calculated as

$$n_S \equiv 1 + \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k} = 1 - (m_1^2 + m_2^2)\,.\tag{4.4}$$

Thus our model predicts $n_S < 1$. As for the tensor-to-scalar ratio, it is given by

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S} = 8 \frac{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^2}{g_1^2 \cos^2\gamma + g_2^2 \sin^2\gamma}\,.\tag{4.5}$$

As we see from Eqs. (4.3), (4.4) and (4.5), our model seems to have a sufficient number of parameters that can be tuned to give the values of n_S and r which are consistent with observations. This is in contrast to a single-field hybrid inflation model with $V = V_0\, e^{m\phi}$, for which the number of e-folds is given by

$$N_{\mathrm{single}} = \frac{\phi - \phi_f}{m}\,,\tag{4.6}$$

which gives

$$n_S = 1 - m^2\,,\quad r = 8m^2 \quad \text{(single-field case)}\,,\tag{4.7}$$

and, to first order in the slow-roll approximation, the non-Gaussianity is exactly zero. Comparing this single-field case with our two-field case, it is amazing that adding only one extra component to the inflaton field results in the huge variety of the output parameters.

Now we evaluate the non-Gaussianity in our model. For convenience, we introduce the linear curvature perturbation $\mathcal{R}_L$ and the linear entropy perturbation S,

$$\mathcal{R}_L = \frac{\delta\phi_1\, g_1 \cos\gamma + \delta\phi_2\, g_2 \sin\gamma}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma}\,,\quad S = \frac{\delta\phi_1\, g_2 \sin\gamma - \delta\phi_2\, g_1 \cos\gamma}{m_2 g_1 \cos\gamma - m_1 g_2 \sin\gamma}\,.\tag{4.8}$$

For the Gaussian fluctuations $\delta\phi_A$ given by Eq. (4.2), we see that they are orthogonal to each other:

$$\langle \mathcal{R}_L \cdot S \rangle = 0\,.\tag{4.9}$$

In terms of $\mathcal{R}_L$ and S, the nonlinear δN in Eq. (4.1) is expressed as

$$\delta N = \mathcal{R}_L + \frac{3}{5} f_{NL}^{\mathrm{local}} (\mathcal{R}_L + S)^2\,,\tag{4.10}$$

where

$$f_{NL}^{\mathrm{local}} = \frac{5 g_1^2 g_2^2}{6\sigma (g_1^2 \cos^2\gamma + g_2^2 \sin^2\gamma)^2} \frac{(m_2 g_1 \cos\gamma - m_1 g_2 \sin\gamma)^2}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma}\,.\tag{4.11}$$

Since both $m_1 g_1 \cos\gamma$ and $m_2 g_2 \sin\gamma$ are positive, we thus see that in this model f_{NL}^{local} is always non-negative. In addition, we note that this non-Gaussianity comes essentially from the end of inflation, as seen from its explicit dependence on the parameters that define the end of inflation. This fact may be explicitly seen by separating δN into two parts; one from the perturbation during inflation δN_* and the other from the end of inflation δN_e. This is done in Appendix A.

We also see that the linear entropy perturbation S induces a curvature perturbation at second order. It may be worthwhile to note that if one finds a way to extract out observationally the second order term, it may be possible to detect this second order effect of the entropy perturbation. In this connection, we mention recent work by Enqvist et al. [21] and by Chambers and Rajantie [10], in which they discussed a mechanism of generating non-Gaussianity at the end of inflation through parametric resonance when there exist two light scalar fields. Their mechanism corresponds effectively to the case where the second order term in (4.10) is dominated by S^2. In this case, the curvature perturbation may be approximately expressed as

$$\delta N = \mathcal{R}_L + f_S S^2 \,. \tag{4.12}$$

Chambers and Rajantie [10] define a non-Gaussian parameter essentially by the ratio,

$$f_{NL}^{\text{CR}} \sim \frac{\left\langle \delta N^3 \right\rangle}{\left\langle \delta N^2 \right\rangle^2} \,. \tag{4.13}$$

Then, under the assumption that $\mathcal{R}_L$ dominates over $f_S S^2$ in δN, that is, $f_S^2 \left\langle S^2 \right\rangle^2 \ll \left\langle \mathcal{R}_L^2 \right\rangle$, one finds

$$f_{NL}^{\text{CR}} \sim f_S^3 \frac{\left\langle S^2 \right\rangle^3}{\left\langle \mathcal{R}_L^2 \right\rangle^2} \,. \tag{4.14}$$

In our case, since $f_S \sim f_{NL}^{\text{local}}$ and $\left\langle S^2 \right\rangle \sim \left\langle \mathcal{R}_L^2 \right\rangle$, our f_{NL}^{CR} would be small $\sim (f_{NL}^{\text{local}})^3 \left\langle S^2 \right\rangle$ even for $f_{NL}^{\text{local}} \sim 100$.

To examine the viability of our model, let us compare our predictions with observations. Analysis of the WMAP 5-year data [22, 23] gives the spectral index of the value,

$$n_S = 0.96 \, {}^{+0.014}_{-0.015} \,, \tag{4.15}$$

and the upper limit on the tensor-to-scalar ratio,

$$\begin{aligned} r &< 0.43 \quad (95\% \text{ CL: WMAP alone}) \,, \\ r &< 0.20 \quad (95\% \text{ CL: WMAP+BAO+SN}) \,. \end{aligned} \tag{4.16}$$

As for the non-Gaussian parameter f_{NL}^{local}, the situation is somewhat controversial. Yadav and Wandelt [24] finds non-vanishing non-Gaussianity in the 3-year WMAP data [25] as

$$27 < f_{NL}^{\text{local}} < 147 \quad (95\% \text{ CL}) \,, \tag{4.17}$$

while Komatsu et al. finds only a bound in the WMAP 5-year data [23],

$$-9 < f_{NL}^{\text{local}} < 111 \quad (95\% \text{ CL}) \,. \tag{4.18}$$

In any case, these results indicate that a positive value of f_{NL}^{local} is more favored over a negative value, if non-zero. This is perfectly consistent with our result, Eq. (4.11).

To show that these observational data can be indeed reproduced in our model, let us consider a specific set of the model parameters. We set

$$m_1^2 \sim 0.005 \,, \quad m_2^2 \sim 0.035 \,, \tag{4.19}$$

and assume that the inflationary trajectory satisfies

$$m_1 \cos\gamma \gg m_2 \sin\gamma \,, \tag{4.20}$$

that is, $\gamma \ll 1$. We also assume that the coupling constants g_1 and g_2 are of the same order of magnitude. In this case, n_S and r depend only on m_1 and m_2,

$$n_S = 1 - (m_1^2 + m_2^2) \sim 0.96 \,, \quad r \approx 8 m_1^2 \sim 0.04 \,. \tag{4.21}$$

On the other hand, the non-Gaussian parameter $f_{NL}^{\rm local}$ depends on the other model parameters. For simplicity, let us assume

$$g_1^2 = g_2^2 \equiv g^2 \,. \tag{4.22}$$

Then the curvature perturbation spectrum is approximately given by

$$\mathcal{P}_S \approx \frac{1}{m_1^2} \left(\frac{H}{2\pi} \right)^2 \sim 2.5 \times 10^{-9} \,, \tag{4.23}$$

where the second equality is from the WMAP normalization [26]. Since we have fixed m_1^2 already, this normalization determines H^2, and hence σ^4 as a function of λ,

$$H^2 \sim 40 \, m_1^2 \mathcal{P}_S \sim 5 \times 10^{-10} \,, \quad \sigma^2 = (12\lambda H^2)^{1/2} \sim \lambda^{1/2} \times 10^{-4} \,. \tag{4.24}$$

This gives

$$f_{NL}^{\rm local} \approx \frac{5 \, g \, m_2^2}{6 \, m_1 \sigma} \sim 40 \, \frac{g}{\lambda^{1/4}} \,. \tag{4.25}$$

Thus, in particular, the non-Gaussian parameter $f_{NL}^{\rm local}$ can be large if λ is very small. Note, however, that λ cannot be too small. For $\sigma \gg H$ which is necessary for the field χ to work as a water-fall field, we must have $\lambda^{1/2} \gg 10^{-6}$ from Eqs. (4.24), hence $\lambda^{1/4} \gg 10^{-3}$.

V. CONCLUSION

We have investigated analytically the curvature perturbation from a model of multi-field hybrid inflation, which we call "*multi-brid inflation*".

First we have considered a general condition for a model to be exactly soluble under the slow-roll approximation. Then we have presented a multi-brid inflation model in which the curvature perturbation is analytically computable to full nonlinear order. We have described a method to calculate the curvature perturbation analytically. Here we have noted that the coupling of the (multi-component) inflaton field to a water-fall field that terminates inflation plays a significant role in the determination of the curvature perturbation.

Then as a specific example, we have focused on a two-brid inflation model, and derived an analytical expression for the curvature perturbation to second order in the field fluctuations expressed in terms of the model parameters. Using this expression, we have given analytical formulas for the spectrum of the curvature perturbation $\mathcal{P}_S$, the spectral index n_S, the tensor-to-scalar ratio r, and the non-Gaussian parameter $f_{NL}^{\rm local}$. We have noted that adding only one extra component to the inflaton field increases substantially the degrees of freedom in the output parameters that are observationally constrained. We have found that our two-brid model can explain the recent WMAP observations [22, 23, 25].

One possibly subtle issue we have not discussed in this paper is the effect of isocurvature perturbations at or after reheating. When the universe is reheated by the water-fall field (and perhaps by the inflaton field as well), the abundance of created particles may depend on the initial values of the components of the inflaton field. This can be regarded as a two-field version of the modulated fluctuations [6, 7]. This would lead to additional power to the curvature perturbation spectrum and probably to additional non-Gaussianity. This is an interesting effect which deserves further study. But from the point of view of the model we discussed in this paper, this dependence must be weak enough for our model to be viable. This effect must be carefully examined when we actually attempt to carry out the model-construction. In fact, Barnaby and Cline found that the effect is significant and leads to non-trivial constraints on some models of hybrid inflation [27].

Recently a lot of efforts have been paid in the construction of stringy inflation models (see e.g. [28, 29] and references therein; for reviews on stringy cosmology, see e.g. [30]). Many of these models fall into a class called brane inflation [31], in which a brane is attracted to another brane (or anti-brane) in higher dimensions and the distance between the branes play the role of the inflaton. In these models, inflation ends with a collision of the branes. Then the collided branes annihilate to heat up the universe. Thus, in the 4-dimensional effective picture, brane inflation is the same as hybrid inflation. Therefore it seems quite possible to construct a stringy model that gives rise to multi-brid inflation. This is currently under investigation.

APPENDIX A: CALCULATION OF δN TO SECOND ORDER

Let us first evaluate the perturbation in γ to second order. Setting $\delta\gamma = \delta_1\gamma + \delta_2\gamma$, where $\delta_1\gamma$ and $\delta_2\gamma$ are of linear and second orders, respectively, we take the perturbation of Eq. (3.11) to second order, assuming $\delta\phi_1$ and $\delta\phi_2$ are of linear order. We have

$$\frac{\delta\phi_1}{m_1} - \frac{\delta\phi_2}{m_2} = -\sigma\left(\frac{\sin\gamma}{g_1 m_1} + \frac{\cos\gamma}{g_2 m_2}\right)(\delta_1\gamma + \delta_2\gamma) - \frac{\sigma}{2}\left(\frac{\cos\gamma}{g_1 m_1} - \frac{\sin\gamma}{g_2 m_2}\right)(\delta_1\gamma)^2. \tag{A1}$$

The linear part of the above equation determines $\delta_1\gamma$. We find

$$\delta_1\gamma = -\frac{g_1 g_2}{\sigma}\frac{m_2\delta\phi_1 - m_1\delta\phi_2}{g_1 m_1\cos\gamma + g_2 m_2\sin\gamma}. \tag{A2}$$

Then collecting the second order terms in Eq. (A1), we find

$$\begin{aligned}
\delta_2\gamma &= \frac{1}{2}\frac{g_1 m_1\sin\gamma - g_2 m_2\cos\gamma}{g_1 m_1\cos\gamma + g_2 m_2\sin\gamma}(\delta_1\gamma)^2 \\
&= \frac{g_1^2 g_2^2}{2\sigma^2}\frac{g_1 m_1\sin\gamma - g_2 m_2\cos\gamma}{(g_1 m_1\cos\gamma + g_2 m_2\sin\gamma)^3}(m_2\delta\phi_1 - m_1\delta\phi_2)^2.
\end{aligned} \tag{A3}$$

Now it is straightforward to calculate δN. Expanding eq. (3.12) to second order in the perturbation, and substituting Eqs. (A2) and (A3) into the resulting equation, we obtain

$$\delta N = \frac{\delta\phi_1\,g_1\cos\gamma + \delta\phi_2\,g_2\sin\gamma}{m_1 g_1\cos\gamma + m_2 g_2\sin\gamma} + \frac{g_1^2 g_2^2}{2\sigma}\frac{(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1 g_1\cos\gamma + m_2 g_2\sin\gamma)^3}. \tag{A4}$$

This is recapitulated in Eq. (4.1).

It may be noted that as long as explicit computations of δN are concerned, it is easier to go back to the original slow-roll equations (3.2). Integrating them are trivial. We immediately find

$$\frac{\phi_1 - \phi_{1,f}}{m_1} = N, \quad \frac{\phi_2 - \phi_{2,f}}{m_1} = N, \tag{A5}$$

where $\phi_{1,f}$ and $\phi_{2,f}$ are parameterized in terms of γ as in Eq. (3.10),

$$\phi_{1,f} = \frac{\sigma}{g_1}\cos\gamma, \quad \phi_{2,f} = \frac{\sigma}{g_2}\sin\gamma. \tag{A6}$$

One can then take the perturbation of either the first or the second equation in Eq. (A5) to second order, with $\delta\phi_{1,f}$ and $\delta\phi_{2,f}$ given by

$$\begin{aligned}
\delta\phi_{1,f} &= \frac{\sigma}{g_1}\left[-\sin\gamma\,(\delta_1\gamma + \delta_2\gamma) - \cos\gamma\frac{(\delta_1\gamma)^2}{2}\right], \\
\delta\phi_{2,f} &= \frac{\sigma}{g_1}\left[\cos\gamma\,(\delta_1\gamma + \delta_2\gamma) - \sin\gamma\frac{(\delta_1\gamma)^2}{2}\right].
\end{aligned} \tag{A7}$$

The result is the same whatever method one adopts. In fact this may be used to check the calculation. For example, taking the perturbation of the first equation in (A5) and using the first line of Eq. (A7), one finds

$$\delta N = \frac{\delta\phi_1}{m_1} - \frac{\delta\phi_{1,f}}{m_1} = \frac{\delta\phi_1}{m_1} + \frac{\sigma}{m_1 g_1}\sin\gamma\,\delta_1\gamma + \frac{\sigma}{m_1 g_1}\left[\sin\gamma\,\delta_2\gamma + \cos\gamma\frac{(\delta_1\gamma)^2}{2}\right]. \tag{A8}$$

Inserting Eqs. (A2) and (A3) into this equation, one can check that the result agrees with Eq. (A4).

Now let us turn to the evaluation of the correction term after inflation, δN_c. Taking the perturbation of N_c in Eq. (3.15), we find

$$\begin{aligned}
\delta N_c &= \frac{1}{4}\left[m_1 \delta\phi_{1,f} + m_2 \delta\phi_{2,f}\right] = -\frac{\sigma}{4}\left(\frac{m_1}{g_1}\sin\gamma - \frac{m_2}{g_2}\cos\gamma\right)\delta_1\gamma \\
&\quad -\frac{\sigma}{4}\left[\left(\frac{m_1}{g_1}\sin\gamma - \frac{m_2}{g_2}\cos\gamma\right)\delta_2\gamma + \left(\frac{m_1}{g_1}\cos\gamma + \frac{m_2}{g_2}\sin\gamma\right)\frac{(\delta_1\gamma)^2}{2}\right].
\end{aligned} \tag{A9}$$

Inserting $\delta_1\gamma$ and $\delta_2\gamma$ given by Eqs. (A2) and (A3) into the above, we obtain

$$\delta N_c = \frac{1}{4}\frac{m_1 g_2 \sin\gamma - m_2 g_1 \cos\gamma}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma}(m_2\delta\phi_1 - m_1\delta\phi_2) - \frac{g_1^2 g_2^2}{8\sigma}\frac{(m_1^2 + m_2^2)(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^3}. \tag{A10}$$

Comparing this with Eq. (A4), we see that the contribution of δN_c to δN is suppressed by a factor quadratic in m_1 and/or m_2. Hence, under the assumption that m_1 and m_2 are sufficiently smaller than unity, which is necessary for the slow-roll condition to hold, we may neglect the correction term δN_c.

Finally, let us evaluate δN up to a surface of constant potential $V = \text{const.} = V_*$ during inflation, which we denote by δN_*, and identify the contribution from the end of inflation, which we denote by δN_e. On superhorizon scales, and uder the slow-roll approximation, δN_* is equal to the curvature perturbation on the uniform density slice, which is usually denoted by ζ.

The slice $V = V_*$ is given by

$$m_1\phi_{1,*} + m_2\phi_{2,*} = \ln(V_*/V_0) \equiv C. \tag{A11}$$

One can parametrize $\phi_{A,*}$ ($A = 1, 2$) with a parameter t as

$$\phi_{1,*} = \frac{1}{2m_1}(C + 2t), \quad \phi_{2,*} = \frac{1}{2m_2}(C - 2t). \tag{A12}$$

This gives

$$\delta\phi_{1,*} = \frac{1}{m_1}\delta t, \quad \delta\phi_{2,*} = -\frac{1}{m_2}\delta t. \tag{A13}$$

Thus taking the perturbation of the solutions (A5), with $\phi_{A,f}$ replaced by $\phi_{A,*}$ and N by N_*, we obtain two expressions for δN_*,

$$\begin{aligned}
\delta N_* &= \frac{\delta\phi_1}{m_1} - \frac{\delta\phi_{1,*}}{m_1} = \frac{\delta\phi_1}{m_1} - \frac{\delta t}{m_1^2}, \\
\delta N_* &= \frac{\delta\phi_2}{m_2} - \frac{\delta\phi_{2,*}}{m_2} = \frac{\delta\phi_2}{m_2} + \frac{\delta t}{m_2^2}.
\end{aligned} \tag{A14}$$

From these equations, we find

$$\delta t = \frac{m_1 m_2}{m_1^2 + m_2^2}(m_2\delta\phi_1 - m_1\delta\phi_2). \tag{A15}$$

Inserting this to either one of Eqs. (A14), we obtain

$$\delta N_* = \frac{m_1\delta\phi_1 + m_2\delta\phi_2}{m_1^2 + m_2^2}. \tag{A16}$$

As clear from this expression, in our model, there is no non-Gaussianity from the evolution during inflation. This is in agreement with the discussion based on the equations in the q_A space given near the end of Section III.

Subtracting δN_* from δN given in Eq. (A4), we obtain the contribution from the end of inflation δN_e. The result is

$$\begin{aligned}
\delta N_e &= \delta N - \delta N_* \\
&= \frac{(m_2 g_1 \cos\gamma - m_1 g_2 \sin\gamma)(m_2\delta\phi_1 - m_1\delta\phi_2)}{(m_1^2 + m_2^2)(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)} + \frac{g_1^2 g_2^2}{2\sigma}\frac{(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^3}.
\end{aligned} \tag{A17}$$

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